ΛΥΜΕΝΕΣ ΑΣΚΗΣΕΙΣ ΣΤΟΝ ΠΟΛΛΑΠΛΑΣΙΑΣΜΟ ΠΡΑΓΜΑΤΙΚΟΥ ΑΡΙΘΜΟΥ ΕΠΙ ΔΙΑΝΥΣΜΑ

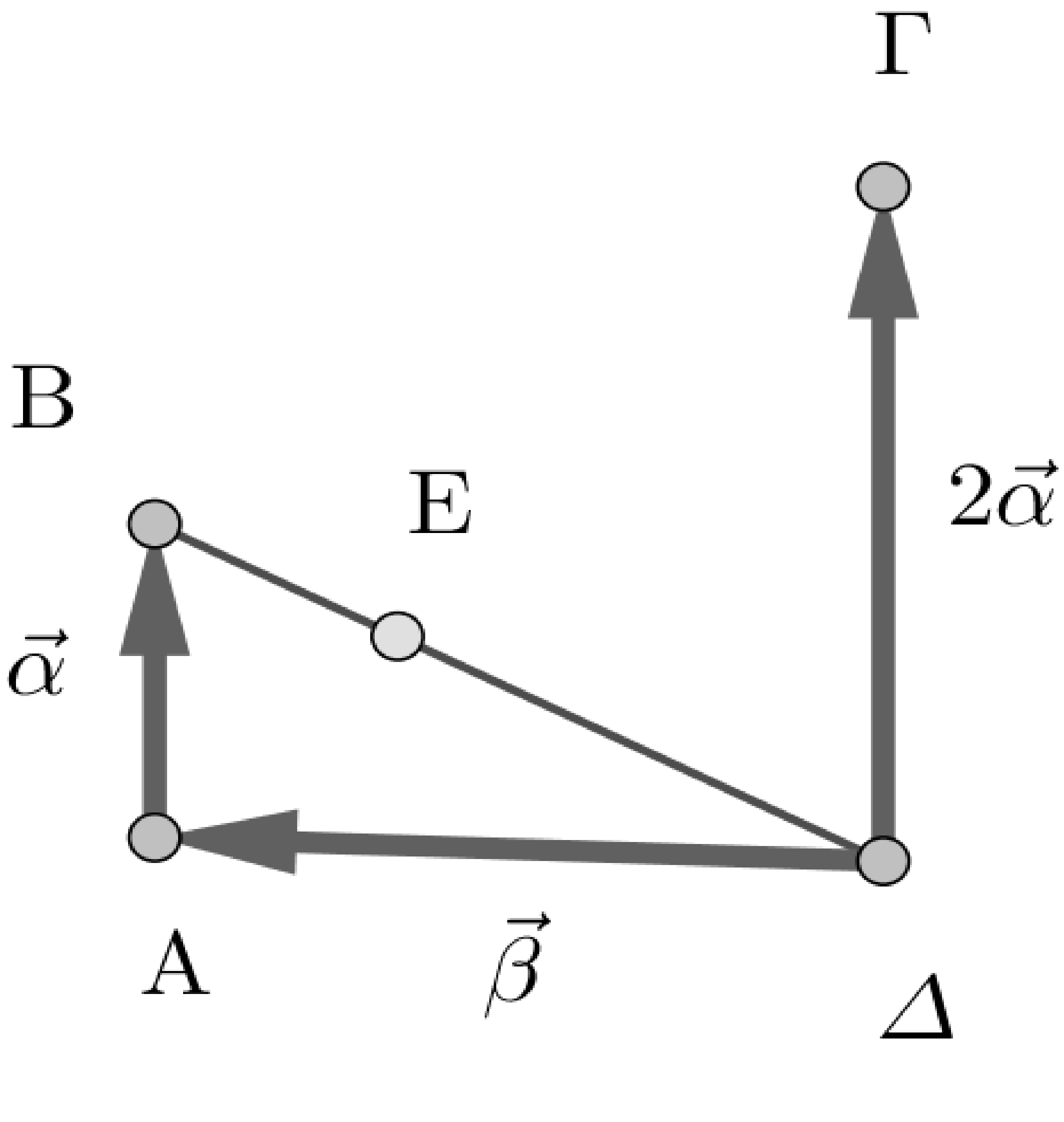
1.

$\Sigma\tau o\ \pi\alpha\rho\alpha\kappa\acute{\alpha}\tau\omega\ \sigma\chi\acute{\eta}\mu\alpha\ \acute{\varepsilon}\chi o\upsilon\mu\varepsilon:$

$\Delta\mathrm{E} = 2\mathrm{EB},\ \overrightarrow{\mathrm{AB}} = \vec{\alpha},\ \overrightarrow{\Delta\Gamma} = 2\vec{\alpha}\ \kappa\alpha\iota\ \overrightarrow{\Delta\mathrm{A}} = \vec{\beta}$

(I) Να εκφράσετε συναρτήσει των $\vec{\alpha}$ και $\vec{\beta}$ τα διανύσματα $\overrightarrow{\Delta\mathrm{B}},\ \overrightarrow{\mathrm{EB}},\ \overrightarrow{\Gamma\mathrm{B}},\ \overrightarrow{\mathrm{AE}}$ και $\overrightarrow{\mathrm{E}\Gamma}$

(II) Να αποδείξετε ότι τα σημεία Α, Ε και Γ είναι συνευθειακά



$$(\mathrm{I})\ \overrightarrow{\Delta\mathrm{A}}=\vec{\beta}\Leftrightarrow-\overrightarrow{\mathrm{A}\Delta}=\vec{\beta}\Leftrightarrow\overrightarrow{\mathrm{A}\Delta}=-\vec{\beta}$$

$$\overrightarrow{\Delta\Gamma}=2\vec{\alpha}\Leftrightarrow\overrightarrow{\mathrm{A}\Gamma}-\overrightarrow{\mathrm{A}\Delta}=2\vec{\alpha}\overset{\overrightarrow{\mathrm{A}\Delta}=-\vec{\beta}}{\Longleftrightarrow}\overrightarrow{\mathrm{A}\Gamma}-\left(-\vec{\beta}\right)=2\vec{\alpha}\Leftrightarrow\overrightarrow{\mathrm{A}\Gamma}+\vec{\beta}=2\vec{\alpha}$$

$$\Leftrightarrow\overrightarrow{\mathrm{A}\Gamma}=2\vec{\alpha}-\vec{\beta}$$

$$\overrightarrow{\Delta\mathrm{E}}=2\overrightarrow{\mathrm{EB}}\Leftrightarrow\overrightarrow{\mathrm{AE}}-\overrightarrow{\mathrm{A}\Delta}=2\left(\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{AE}}\right)\overset{\substack{\overrightarrow{\mathrm{A}\Delta}=-\vec{\beta}\\ \overrightarrow{\mathrm{AB}}=\vec{\alpha}}}{\Longleftrightarrow}$$

$$\overrightarrow{\mathrm{AE}}-\left(-\vec{\beta}\right)=2\left(\vec{\alpha}-\overrightarrow{\mathrm{AE}}\right)\Leftrightarrow\overrightarrow{\mathrm{AE}}+\vec{\beta}=2\vec{\alpha}-2\overrightarrow{\mathrm{AE}}\Leftrightarrow$$

$$\overrightarrow{\mathrm{AE}}+2\overrightarrow{\mathrm{AE}}=2\vec{\alpha}-\vec{\beta}\Leftrightarrow3\overrightarrow{\mathrm{AE}}=2\vec{\alpha}-\vec{\beta}\Leftrightarrow\overrightarrow{\mathrm{AE}}=\frac{2\vec{\alpha}-\vec{\beta}}{3}$$

$$\mathrm{O}\pi\acute{o}\tau\varepsilon:\boxed{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\overrightarrow{\mathrm{A}\Delta}=-\vec{\beta},\overrightarrow{\mathrm{A}\Gamma}=2\vec{\alpha}-\vec{\beta},\overrightarrow{\mathrm{AE}}=\frac{2\vec{\alpha}-\vec{\beta}}{3}}$$

$$\overrightarrow{\Delta\mathrm{B}}=\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{A}\Delta}\overset{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\overrightarrow{\mathrm{A}\Delta}=-\vec{\beta}}{=}\vec{\alpha}-\left(-\vec{\beta}\right)=\vec{\alpha}+\vec{\beta}$$

$$\overrightarrow{\mathrm{EB}}=\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{AE}}\overset{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\overrightarrow{\mathrm{AE}}=\frac{2\vec{\alpha}-\vec{\beta}}{3}}{=}\vec{\alpha}-\frac{2\vec{\alpha}-\vec{\beta}}{3}=\frac{3\vec{\alpha}-\left(2\vec{\alpha}-\vec{\beta}\right)}{3}=$$

$$=\frac{3\vec{\alpha}-2\vec{\alpha}+\vec{\beta}}{3}=\frac{\vec{\alpha}+\vec{\beta}}{3}$$

$$\overrightarrow{\Gamma\mathrm{B}}=\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{A}\Gamma}\overset{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\overrightarrow{\mathrm{A}\Gamma}=2\vec{\alpha}-\vec{\beta}}{=}\vec{\alpha}-\left(2\vec{\alpha}-\vec{\beta}\right)=\vec{\alpha}-2\vec{\alpha}+\vec{\beta}=\vec{\beta}-\vec{\alpha}$$

$$\overrightarrow{\mathrm{E}\Gamma}=\overrightarrow{\mathrm{A}\Gamma}-\overrightarrow{\mathrm{AE}}\overset{\overrightarrow{\mathrm{A}\Gamma}=2\vec{\alpha}-\vec{\beta},\overrightarrow{\mathrm{AE}}=\frac{2\vec{\alpha}-\vec{\beta}}{3}}{=}2\vec{\alpha}-\vec{\beta}-\frac{2\vec{\alpha}-\vec{\beta}}{3}=$$

$$=\frac{3\left(2\vec{\alpha}-\vec{\beta}\right)}{3}-\frac{2\vec{\alpha}-\vec{\beta}}{3}=\frac{6\vec{\alpha}-3\vec{\beta}-\left(2\vec{\alpha}-\vec{\beta}\right)}{3}=\frac{6\vec{\alpha}-3\vec{\beta}-2\vec{\alpha}+\vec{\beta}}{3}$$

$$=\frac{4\vec{\alpha}-2\vec{\beta}}{3}=\frac{2}{3}\left(2\vec{\alpha}-\vec{\beta}\right)$$

$$(\mathrm{II})\ \overrightarrow{\mathrm{E\Gamma}}=\frac{2}{3}\left(2\vec{\alpha}-\vec{\beta}\right)=2\,\frac{2\vec{\alpha}-\vec{\beta}}{3}\overset{\overrightarrow{\mathrm{AE}}=\frac{2\vec{\alpha}-\vec{\beta}}{3}}{=}2\overrightarrow{\mathrm{AE}}$$

Επειδή $\overrightarrow{\mathrm{E\Gamma}}=2\overrightarrow{\mathrm{AE}}$ *προκύπτει ότι* $\overrightarrow{\mathrm{E\Gamma}}\,//\,\overrightarrow{\mathrm{AE}}$. *Οπότε τα διανύσματα βρίσκονται στον ίδιο φορέα ή σε παράλληλους φορείς. Αν τα διανύσματα* $\overrightarrow{\mathrm{E\Gamma}},\overrightarrow{\mathrm{AE}}$ *βρίσκονται σε παράλλληλους φορείς θα έχω* $\mathrm{E\Gamma}\,//\,\mathrm{AE}$. *Άτοπο γιατί οι ευθείες* $\mathrm{E\Gamma}$ *και* AE *είναι παράλληλες και έχουν ένα κοινό σημείο*!!! *Συνεπώς τα διανύσματα* $\overrightarrow{\mathrm{E\Gamma}},\overrightarrow{\mathrm{AE}}$ *έχουν τον ίδιο φορέα. Άρα τα σημεία* $\mathrm{A,E,\Gamma}$ *είναι συνευθειακά.*

2.

Αν $\overrightarrow{\mathrm{AK}}+3\overrightarrow{\mathrm{BK}}-2\overrightarrow{\mathrm{BA}}=\overrightarrow{\mathrm{B\Lambda}}+3\overrightarrow{\mathrm{AM}}$, *να αποδείξετε οτι τα σημεία* $\mathrm{K,\Lambda}$ *και* M *είναι συνευθειακά.*

Έχω: $\boxed{\overrightarrow{\mathrm{BK}}=\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{AB}},\overrightarrow{\mathrm{B\Lambda}}=\overrightarrow{\mathrm{A\Lambda}}-\overrightarrow{\mathrm{AB}}}$

$$\overrightarrow{\mathrm{AK}}+3\overrightarrow{\mathrm{BK}}-2\overrightarrow{\mathrm{BA}}=\overrightarrow{\mathrm{B\Lambda}}+3\overrightarrow{\mathrm{AM}}\overset{\substack{\overrightarrow{\mathrm{BK}}=\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{AB}},\overrightarrow{\mathrm{BA}}=-\overrightarrow{\mathrm{AB}}\\ \overrightarrow{\mathrm{B\Lambda}}=\overrightarrow{\mathrm{A\Lambda}}-\overrightarrow{\mathrm{AB}}}}{\Longleftrightarrow}$$

$$\overrightarrow{\mathrm{AK}}+3\left(\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{AB}}\right)-2\left(-\overrightarrow{\mathrm{AB}}\right)=\overrightarrow{\mathrm{A\Lambda}}-\overrightarrow{\mathrm{AB}}+3\overrightarrow{\mathrm{AM}}\Leftrightarrow$$

$$\overrightarrow{\mathrm{AK}}+3\overrightarrow{\mathrm{AK}}-3\overrightarrow{\mathrm{AB}}+2\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{A\Lambda}}-\overrightarrow{\mathrm{AB}}+3\overrightarrow{\mathrm{AM}}\Leftrightarrow$$

$$\overrightarrow{\mathrm{AK}}+3\overrightarrow{\mathrm{AK}}-\cancel{\overrightarrow{\mathrm{AB}}}=\overrightarrow{\mathrm{A\Lambda}}-\cancel{\overrightarrow{\mathrm{AB}}}+3\overrightarrow{\mathrm{AM}}\Leftrightarrow$$

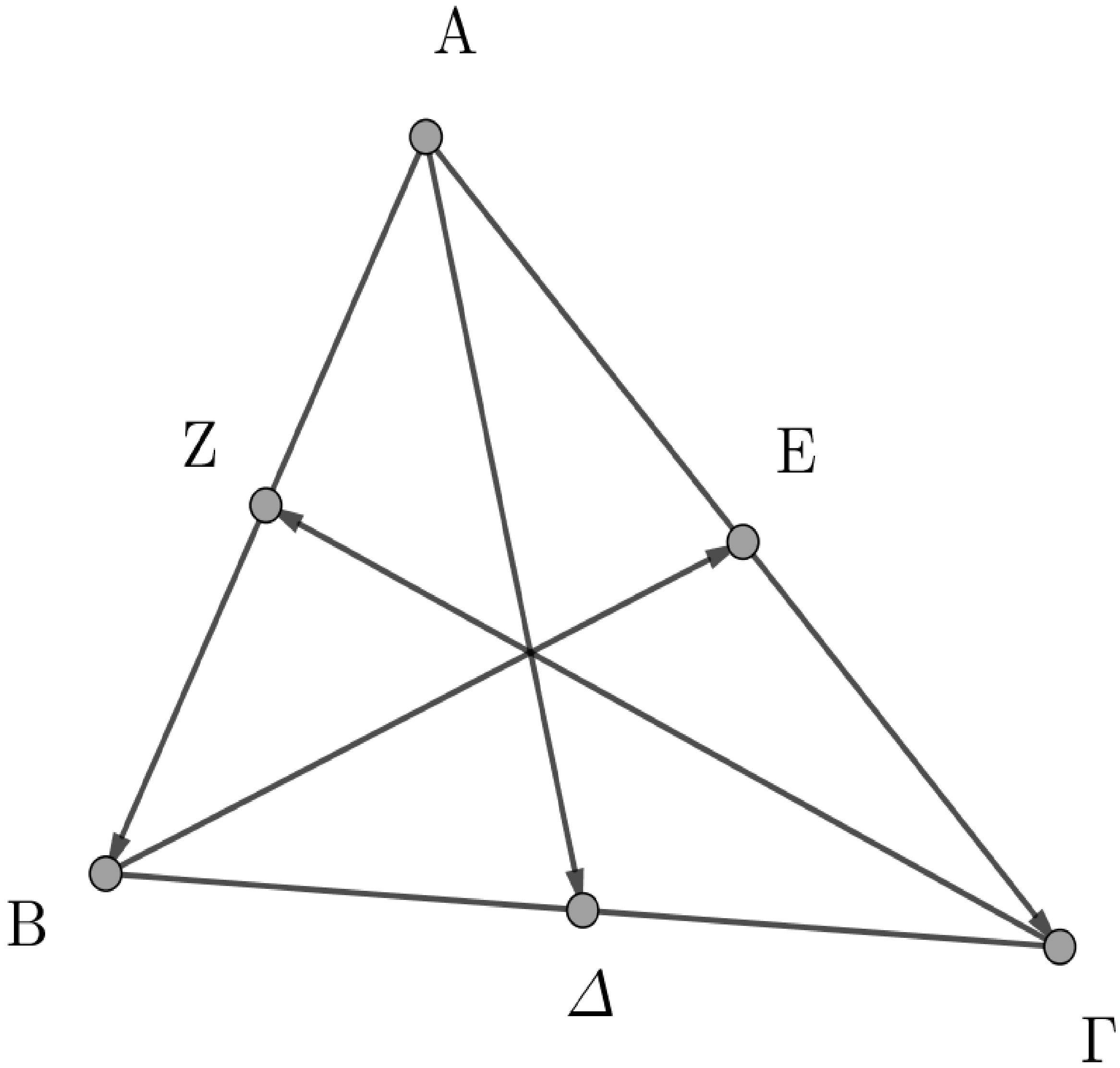
$$\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{A\Lambda}}=3\overrightarrow{\mathrm{AM}}-3\overrightarrow{\mathrm{AK}}\overset{\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{A\Lambda}}=\overrightarrow{\mathrm{\Lambda K}}}{\Longleftrightarrow}\overrightarrow{\mathrm{\Lambda K}}=3\left(\overrightarrow{\mathrm{AM}}-\overrightarrow{\mathrm{AK}}\right)$$

$$\overset{\overrightarrow{\mathrm{AM}}-\overrightarrow{\mathrm{AK}}=\overrightarrow{\mathrm{KM}}}{\Longleftrightarrow}\overrightarrow{\mathrm{\Lambda K}}=3\overrightarrow{\mathrm{KM}}\Rightarrow\overrightarrow{\mathrm{\Lambda K}}\,//\,\overrightarrow{\mathrm{KM}}$$

Οπότε τα σημεία $\mathrm{K,\Lambda,M}$ *είναι συνευθειακά.*

3.

Αν $\mathrm{A\Delta,BE,\Gamma Z}$ *είναι οι διάμεσοι τριγώνου* $\mathrm{AB\Gamma}$, *να αποδείξετε ότι*: $\overrightarrow{\mathrm{A\Delta}}+\overrightarrow{\mathrm{BE}}+\overrightarrow{\mathrm{\Gamma Z}}=\vec{0}$



$\Theta\acute{\varepsilon}\tau\omega: \boxed{\overrightarrow{\mathrm{AB}} = \vec{\alpha}, \overrightarrow{\mathrm{A\Gamma}} = \vec{\beta}}$

Επειδή Δ μέσο της ΒΓ θα έχω:

$$\overrightarrow{\mathrm{A\Delta}} = \frac{\overrightarrow{\mathrm{AB}} + \overrightarrow{\mathrm{A\Gamma}}}{2} \overset{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\ \overrightarrow{\mathrm{A\Gamma}}=\vec{\beta}}{\Longrightarrow} \boxed{\overrightarrow{\mathrm{A\Delta}} = \frac{\vec{\alpha} + \vec{\beta}}{2}}$$

Επειδή Ε μέσο της ΒΓ θα έχω:

$$\overrightarrow{\mathrm{AE}} = \frac{\overrightarrow{\mathrm{A\Gamma}}}{2} \overset{\overrightarrow{\mathrm{A\Gamma}}=\vec{\beta}}{\Longrightarrow} \boxed{\overrightarrow{\mathrm{AE}} = \frac{\vec{\beta}}{2}}$$

Επειδή Ζ μέσο της ΑΒ θα έχω:

$$\overrightarrow{\mathrm{AZ}} = \frac{\overrightarrow{\mathrm{AB}}}{2} \overset{\overrightarrow{\mathrm{AB}}=\vec{\alpha}}{\Longrightarrow} \boxed{\overrightarrow{\mathrm{AZ}} = \frac{\vec{\alpha}}{2}}$$

$$\overrightarrow{\mathrm{BE}}=\overrightarrow{\mathrm{AE}}-\overrightarrow{\mathrm{AB}}\overset{\substack{\overrightarrow{\mathrm{AE}}=\frac{\vec{\beta}}{2}\\ \overrightarrow{\mathrm{AB}}=\vec{\alpha}}}{=}\frac{\vec{\beta}}{2}-\vec{\alpha}$$

$$\mathrm{O}\pi\acute{o}\tau\varepsilon:\boxed{\overrightarrow{\mathrm{BE}}=\frac{\vec{\beta}}{2}-\vec{\alpha}}$$

$$\overrightarrow{\Gamma\mathrm{Z}}=\overrightarrow{\mathrm{AZ}}-\overrightarrow{\mathrm{A}\Gamma}\overset{\substack{\overrightarrow{\mathrm{AZ}}=\frac{\vec{\alpha}}{2}\\ \overrightarrow{\mathrm{A}\Gamma}=\vec{\beta}}}{=}\frac{\vec{\alpha}}{2}-\vec{\beta}$$

$$\mathrm{O}\pi\acute{o}\tau\varepsilon:\boxed{\overrightarrow{\Gamma\mathrm{Z}}=\frac{\vec{\alpha}}{2}-\vec{\beta}}$$

$$\overrightarrow{\mathrm{A}\Delta}+\overrightarrow{\mathrm{BE}}+\overrightarrow{\Gamma\mathrm{Z}}\overset{\substack{\overrightarrow{\mathrm{A}\Delta}=\frac{\vec{\alpha}+\vec{\beta}}{2}\\ \overrightarrow{\mathrm{BE}}=\frac{\vec{\beta}}{2}-\vec{\alpha}\\ \overrightarrow{\Gamma\mathrm{Z}}=\frac{\vec{\alpha}}{2}-\vec{\beta}}}{=}\frac{\vec{\alpha}+\vec{\beta}}{2}+\frac{\vec{\beta}}{2}-\vec{\alpha}+\frac{\vec{\alpha}}{2}-\vec{\beta}=$$

$$\frac{\vec{\alpha}+\vec{\beta}}{2}+\frac{\vec{\beta}}{2}-\frac{2\vec{\alpha}}{2}+\frac{\vec{\alpha}}{2}-\frac{2\vec{\beta}}{2}=\frac{\vec{\alpha}+\vec{\beta}+\vec{\beta}-2\vec{\alpha}+\vec{\alpha}-2\vec{\beta}}{2}=\vec{0}$$

4.

$$\boxed{\begin{array}{l}\mathrm{A}\nu\ \mathrm{K},\Lambda,\mathrm{M}\ \varepsilon\acute{\iota}\nu\alpha\iota\ \tau\alpha\ \mu\acute{\varepsilon}\sigma\alpha\ \tau\omega\nu\ \pi\lambda\varepsilon\upsilon\rho\acute{\omega}\nu\ \mathrm{B}\Gamma,\Gamma\mathrm{A},\mathrm{AB},\\ \alpha\nu\tau\iota\sigma\tau\omicron\acute{\iota}\chi\omega\varsigma,\tau\rho\iota\gamma\acute{\omega}\nu\omicron\upsilon\ \mathrm{AB}\Gamma,\nu\alpha\ \alpha\pi\omicron\delta\varepsilon\acute{\iota}\xi\varepsilon\tau\varepsilon\ \acute{o}\tau\iota\ \gamma\iota\alpha\\ \omicron\pi\omicron\iota\alpha\delta\acute{\eta}\pi\omicron\tau\varepsilon\ \sigma\eta\mu\varepsilon\acute{\iota}\omicron\ \mathrm{O}\ \iota\sigma\chi\acute{\upsilon}\varepsilon\iota:\\ \overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{O}\Gamma}=\overrightarrow{\mathrm{OK}}+\overrightarrow{\mathrm{O}\Lambda}+\overrightarrow{\mathrm{OM}}\end{array}}$$

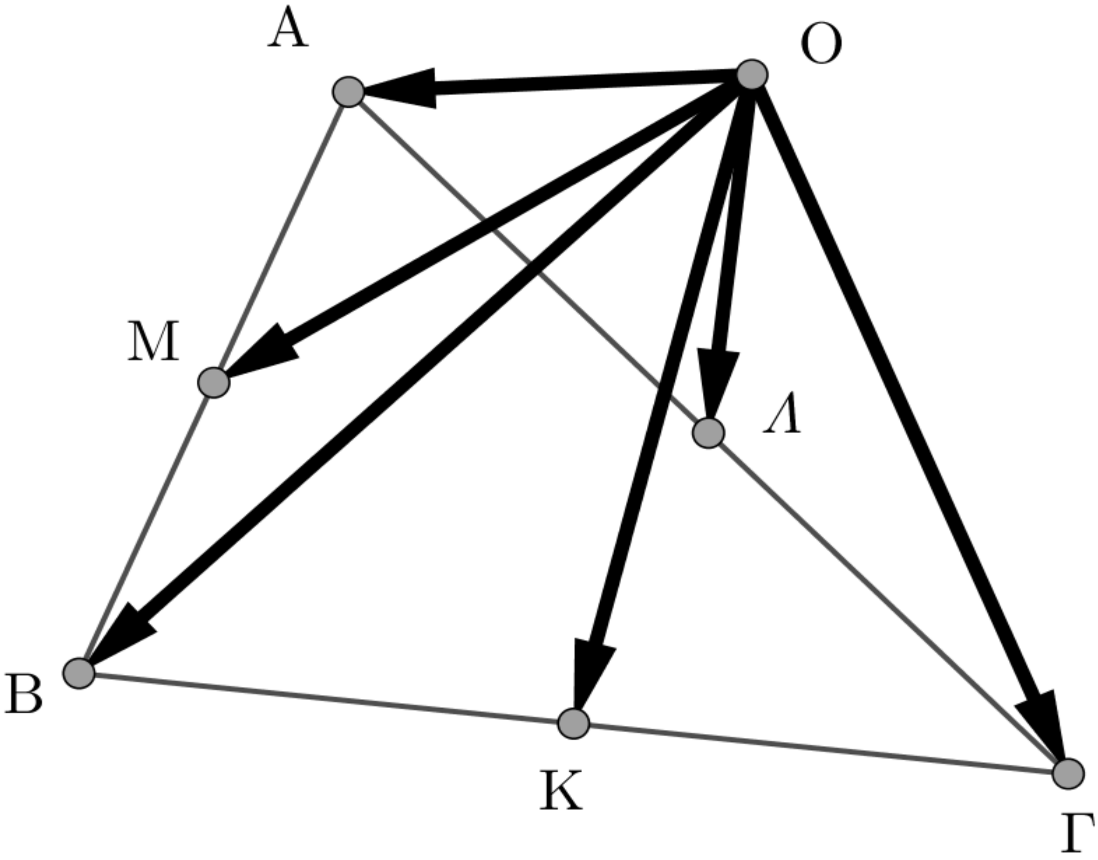
$$\Theta\acute{\varepsilon}\tau\omega:\boxed{\overrightarrow{\mathrm{OA}}=\vec{\alpha},\overrightarrow{\mathrm{OB}}=\vec{\beta},\overrightarrow{\mathrm{O}\Gamma}=\vec{\gamma}}$$

$\mathrm{E}\pi\varepsilon\iota\delta\acute{\eta}\ \mathrm{K}\ \mu\acute{\varepsilon}\sigma\omicron\ \tau\eta\varsigma\ \mathrm{B}\Gamma\ \theta\alpha\ \acute{\varepsilon}\chi\omega:$

$$\overrightarrow{\mathrm{OK}}=\frac{\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{O}\Gamma}}{2}\overset{\substack{\overrightarrow{\mathrm{OB}}=\vec{\beta}\\ \overrightarrow{\mathrm{O}\Gamma}=\vec{\gamma}}}{\Longrightarrow}\boxed{\overrightarrow{\mathrm{OK}}=\frac{\vec{\beta}+\vec{\gamma}}{2}}$$

$\mathrm{E}\pi\varepsilon\iota\delta\acute{\eta}\ \Lambda\ \mu\acute{\varepsilon}\sigma\omicron\ \tau\eta\varsigma\ \Gamma\mathrm{A}\ \theta\alpha\ \acute{\varepsilon}\chi\omega:$

$$\overrightarrow{\mathrm{O}\Lambda}=\frac{\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{O}\Gamma}}{2}\overset{\substack{\overrightarrow{\mathrm{OA}}=\vec{\alpha}\\ \overrightarrow{\mathrm{O}\Gamma}=\vec{\gamma}}}{\Longrightarrow}\boxed{\overrightarrow{\mathrm{O}\Lambda}=\frac{\vec{\alpha}+\vec{\gamma}}{2}}$$



Επειδή Μ *μέσο της* ΑΒ *θα έχω*:

$$\overrightarrow{\mathrm{OM}}=\frac{\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{OB}}}{2}\overset{\substack{\overrightarrow{\mathrm{OA}}=\vec{\alpha}\\ \overrightarrow{\mathrm{O\Gamma}}=\vec{\beta}}}{\Longrightarrow}\boxed{\overrightarrow{\mathrm{OM}}=\frac{\vec{\alpha}+\vec{\beta}}{2}}$$

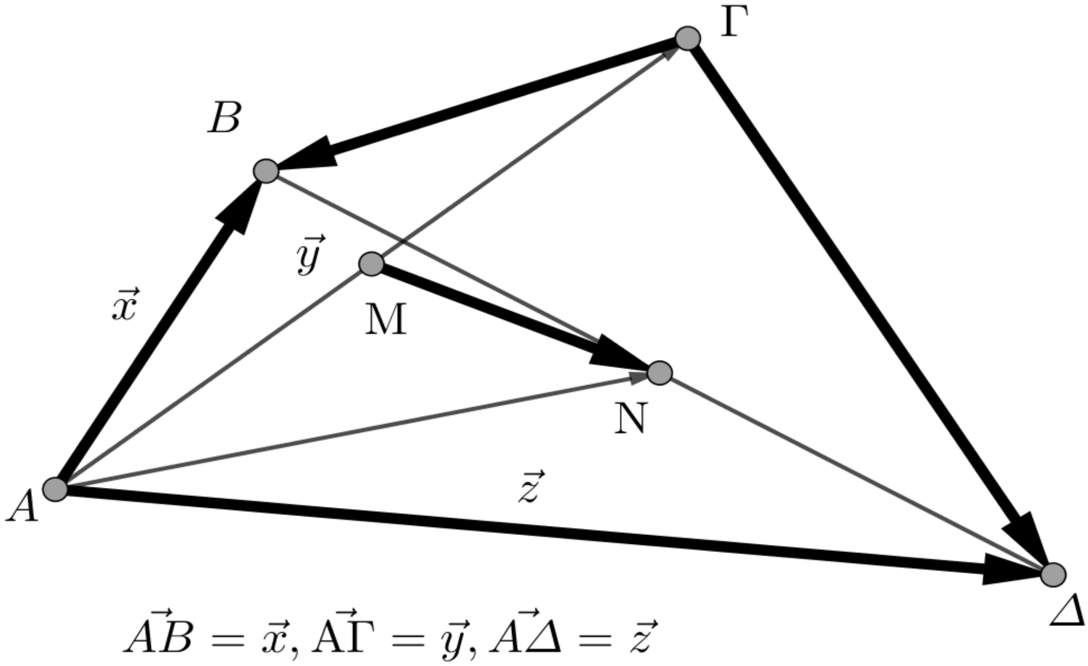
$$\overrightarrow{\mathrm{OK}}+\overrightarrow{\mathrm{O\Lambda}}+\overrightarrow{\mathrm{OM}}\overset{\substack{\overrightarrow{\mathrm{OK}}=\frac{\vec{\beta}+\vec{\gamma}}{2}\\ \overrightarrow{\mathrm{O\Lambda}}=\frac{\vec{\alpha}+\vec{\gamma}}{2}\\ \overrightarrow{\mathrm{OM}}=\frac{\vec{\alpha}+\vec{\beta}}{2}}}{=}\frac{\vec{\beta}+\vec{\gamma}}{2}+\frac{\vec{\alpha}+\vec{\gamma}}{2}+\frac{\vec{\alpha}+\vec{\beta}}{2}=$$

$$\frac{\vec{\beta}+\vec{\gamma}+\vec{\alpha}+\vec{\gamma}+\vec{\alpha}+\vec{\beta}}{2}=\frac{1}{2}\left(2\vec{\alpha}+2\vec{\beta}+2\vec{\gamma}\right)=\frac{1}{2}2\left(\vec{\alpha}+\vec{\beta}+\vec{\gamma}\right)=$$

$$=\vec{\alpha}+\vec{\beta}+\vec{\gamma}\overset{\overrightarrow{\mathrm{OA}}=\vec{\alpha},\overrightarrow{\mathrm{OB}}=\vec{\beta},\overrightarrow{\mathrm{O\Gamma}}=\vec{\gamma}}{=}\overrightarrow{\mathrm{OA}}+\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{O\Gamma}}$$

5.

$\text{Αν Μ και Ν τα μέσα των διαγωνίων ΑΓ και ΒΔ,}$ $\text{αντιστοίχως, ενός τετραπλεύρου ΑΒΓΔ, να αποδείξετε ότι:}$ $\overrightarrow{AB}+\overrightarrow{A\Delta}+\overrightarrow{\Gamma B}+\overrightarrow{\Gamma\Delta}=4\overrightarrow{MN}$



Θέτω: $\boxed{\overrightarrow{AB}=\vec{x},\overrightarrow{A\Gamma}=\vec{y},\overrightarrow{A\Delta}=\vec{z}}$

Επειδή Μ μέσο της ΑΓ θα έχω:

$$\overrightarrow{AM}=\frac{\overrightarrow{A\Gamma}}{2}\overset{\overrightarrow{A\Gamma}=\vec{y}}{\Longrightarrow}\boxed{\overrightarrow{AM}=\frac{\vec{y}}{2}}$$

Επειδή Ν μέσο της ΒΔ θα έχω:

$$\overrightarrow{AN}=\frac{\overrightarrow{AB}+\overrightarrow{A\Delta}}{2}\overset{\substack{\overrightarrow{AB}=\vec{x}\\ \overrightarrow{A\Delta}=\vec{z}}}{\Longrightarrow}\boxed{\overrightarrow{AN}=\frac{\vec{x}+\vec{z}}{2}}$$

$$\overrightarrow{\Gamma B}=\overrightarrow{AB}-\overrightarrow{A\Gamma}\overset{\substack{\overrightarrow{AB}=\vec{x}\\ \overrightarrow{A\Gamma}=\vec{y}}}{\Longrightarrow}\boxed{\overrightarrow{\Gamma B}=\vec{x}-\vec{y}}$$

$$\overrightarrow{\Gamma\Delta}=\overrightarrow{A\Delta}-\overrightarrow{A\Gamma}\overset{\substack{\overrightarrow{A\Delta}=\vec{z}\\ \overrightarrow{A\Gamma}=\vec{y}}}{\Longrightarrow}\boxed{\overrightarrow{\Gamma\Delta}=\vec{z}-\vec{y}}$$

$$\overrightarrow{\mathrm{MN}}=\overrightarrow{\mathrm{AN}}-\overrightarrow{\mathrm{AM}}\overset{\substack{\overrightarrow{\mathrm{AN}}=\frac{\vec{x}+\vec{z}}{2}\\ \overrightarrow{\mathrm{AM}}=\frac{\vec{y}}{2}}}{\Longrightarrow}\overrightarrow{\mathrm{MN}}=\frac{\vec{x}+\vec{z}}{2}-\frac{\vec{y}}{2}\Rightarrow\boxed{\overrightarrow{\mathrm{MN}}=\frac{\vec{x}+\vec{z}-\vec{y}}{2}}$$

$$\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{A\Delta}}+\overrightarrow{\Gamma\mathrm{B}}+\overrightarrow{\Gamma\Delta}\overset{\substack{\overrightarrow{\mathrm{AB}}=\vec{x},\overrightarrow{\mathrm{A\Delta}}=\vec{z}\\ \overrightarrow{\Gamma\mathrm{B}}=\vec{x}-\vec{y},\overrightarrow{\Gamma\Delta}=\vec{z}-\vec{y}}}{=}\vec{x}+\vec{z}+\vec{x}-\vec{y}+\vec{z}-\vec{y}=$$

$$=2\vec{x}-2\vec{y}+2\vec{z}$$

$$\mathrm{O}\pi\acute{o}\tau\varepsilon:\boxed{\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{A\Delta}}+\overrightarrow{\Gamma\mathrm{B}}+\overrightarrow{\Gamma\Delta}=2\vec{x}-2\vec{y}+2\vec{z}}(1)$$

$$4\overrightarrow{\mathrm{MN}}\overset{\overrightarrow{\mathrm{MN}}=\frac{\vec{x}+\vec{z}-\vec{y}}{2}}{=}4\bullet\frac{1}{2}\left(\vec{x}+\vec{z}-\vec{y}\right)=2\left(\vec{x}+\vec{z}-\vec{y}\right)=2\vec{x}+2\vec{z}-2\vec{y}$$

$$\mathrm{O}\pi\acute{o}\tau\varepsilon:\boxed{4\overrightarrow{\mathrm{MN}}=2\vec{x}-2\vec{y}+2\vec{z}}(2)$$

$\mathrm{A}\pi o\ \tau\iota\varsigma\ \sigma\chi\acute{\varepsilon}\sigma\varepsilon\iota\varsigma\ (1),(2)\ \theta\alpha\ \acute{\varepsilon}\chi\omega:$

$$\left.\begin{cases}\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{A\Delta}}+\overrightarrow{\Gamma\mathrm{B}}+\overrightarrow{\Gamma\Delta}=2\vec{x}-2\vec{y}+2\vec{z}\\ 4\overrightarrow{\mathrm{MN}}=2\vec{x}-2\vec{y}+2\vec{z}\end{cases}\right\}\Rightarrow$$

$$\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{A\Delta}}+\overrightarrow{\Gamma\mathrm{B}}+\overrightarrow{\Gamma\Delta}=4\overrightarrow{\mathrm{MN}}$$

6.

$\Delta\acute{\iota}\nu o\nu\tau\alpha\iota\ o\iota\ \sigma\chi\acute{\varepsilon}\sigma\varepsilon\iota\varsigma:$

$$\left.\begin{cases}x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K}\Gamma}=0(1)\\ x\overrightarrow{\Lambda\mathrm{A}}+y\overrightarrow{\Lambda\mathrm{B}}+z\overrightarrow{\Lambda\Gamma}=0(2)\\ x+y+z=0(3)\end{cases}\right\}$$

$\mathrm{A}\nu\ \tau o\ \sigma\eta\mu\varepsilon\acute{\iota}o\ \mathrm{K}\ \varepsilon\acute{\iota}\nu\alpha\iota\ \delta\iota\acute{\alpha}\varphi o\rho o\ \alpha\pi o\ \tau o\ \sigma\eta\mu\varepsilon\acute{\iota}o\ \Lambda\ \nu\alpha$ $\alpha\pi o\delta\varepsilon\acute{\iota}\xi\varepsilon\tau\varepsilon\ \acute{o}\tau\iota\ \alpha\nu\ \iota\sigma\chi\acute{\upsilon}o\upsilon\nu\ \delta\upsilon o\ \alpha\pi o\ \tau\iota\varsigma\ \sigma\chi\acute{\varepsilon}\sigma\varepsilon\iota\varsigma$ $(1),(2),(3)\ \tau\acute{o}\tau\varepsilon\ \theta\alpha\ \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \kappa\alpha\iota\ \eta\ \tau\rho\acute{\iota}\tau\eta.$

$\Gamma\iota\alpha\ \nu\alpha\ \alpha\pi o\delta\varepsilon\acute{\iota}\xi\omega\ \alpha\nu\ \iota\sigma\chi\acute{\upsilon}o\upsilon\nu\ \delta\upsilon o\ \alpha\pi o\ \tau\iota\varsigma\ \sigma\chi\acute{\varepsilon}\sigma\varepsilon\iota\varsigma$ $(1),(2),(3)\ \tau\acute{o}\tau\varepsilon\ \theta\alpha\ \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \kappa\alpha\iota\ \eta\ \tau\rho\acute{\iota}\tau\eta\ \theta\alpha\ \pi\rho\acute{\varepsilon}\pi\varepsilon\iota\ \nu\alpha$ $\delta\varepsilon\acute{\iota}\xi\omega\ \acute{o}\tau\iota:$

$$\left[(1),(2)\Rightarrow(3)\right],\left[(2),(3)\Rightarrow(1)\right],\left[(1),(2)\Rightarrow(3)\right]$$

$\underline{\Theta\alpha\ \alpha\pi o\delta\varepsilon\acute{\iota}\xi\omega\ o\tau\iota\ \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \eta\ \sigma\upsilon\nu\varepsilon\pi\alpha\gamma\omega\gamma\acute{\eta}:(1),(2)\Rightarrow(3)}$

$\mathrm{E}\pi\varepsilon\iota\delta\acute{\eta}\ \iota\sigma\chi\acute{\upsilon}o\upsilon\nu\ o\iota\ (1),(2)\ \theta\alpha\ \acute{\varepsilon}\chi\omega:$

$$\left\{\begin{array}{l} x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0} \\ x\overrightarrow{\Lambda\mathrm{A}}+y\overrightarrow{\Lambda\mathrm{B}}+z\overrightarrow{\Lambda\Gamma}=\vec{0}\end{array}\right\}\Rightarrow\left\{\begin{array}{l} -\left(x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}\right)=\vec{0} \\ -\left(x\overrightarrow{\Lambda\mathrm{A}}+y\overrightarrow{\Lambda\mathrm{B}}+z\overrightarrow{\Lambda\Gamma}\right)=\vec{0}\end{array}\right\}$$

$$\Rightarrow\left\{\begin{array}{l} x\left(-\overrightarrow{\mathrm{KA}}\right)+y\left(-\overrightarrow{\mathrm{KB}}\right)+z\left(-\overrightarrow{\mathrm{K\Gamma}}\right)=\vec{0} \\ x\left(-\overrightarrow{\Lambda\mathrm{A}}\right)+y\left(-\overrightarrow{\Lambda\mathrm{B}}\right)+z\left(-\overrightarrow{\Lambda\Gamma}\right)=\vec{0}\end{array}\right\}\Rightarrow$$

$$\underline{\left\{\begin{array}{l} x\overrightarrow{\mathrm{AK}}+y\overrightarrow{\mathrm{BK}}+z\overrightarrow{\Gamma\mathrm{K}}=\vec{0} \\ x\overrightarrow{\mathrm{A}\Lambda}+y\overrightarrow{\mathrm{B}\Lambda}+z\overrightarrow{\Gamma\Lambda}=\vec{0}\end{array}\right\}}\overset{(-)}{\Longrightarrow}$$

$$x\overrightarrow{\mathrm{AK}}+y\overrightarrow{\mathrm{BK}}+z\overrightarrow{\Gamma\mathrm{K}}-x\overrightarrow{\mathrm{A}\Lambda}-y\overrightarrow{\mathrm{B}\Lambda}-z\overrightarrow{\Gamma\Lambda}=\vec{0}\Rightarrow$$

$$x\left(\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{A}\Lambda}\right)+y\left(\overrightarrow{\mathrm{BK}}-\overrightarrow{\mathrm{B}\Lambda}\right)+z\left(\overrightarrow{\Gamma\mathrm{K}}-\overrightarrow{\Gamma\Lambda}\right)=\vec{0}\ \overset{\substack{\overrightarrow{\mathrm{AK}}-\overrightarrow{\mathrm{A}\Lambda}=\overrightarrow{\Lambda\mathrm{K}}\\ \overrightarrow{\mathrm{BK}}-\overrightarrow{\mathrm{B}\Lambda}=\overrightarrow{\Lambda\mathrm{K}}\\ \overrightarrow{\Gamma\mathrm{K}}-\overrightarrow{\Gamma\Lambda}=\overrightarrow{\Lambda\mathrm{K}}}}{\Longrightarrow}$$

$$x\overrightarrow{\Lambda\mathrm{K}}+y\overrightarrow{\Lambda\mathrm{K}}+z\overrightarrow{\mathrm{K}\Lambda}=\vec{0}\Rightarrow(x+y+z)\overrightarrow{\Lambda\mathrm{K}}=\vec{0}$$

$\mathrm{E}\pi\varepsilon\iota\delta\acute{\eta}\ \Lambda\not\equiv\mathrm{K}\ \acute{\varepsilon}\chi\omega\ \overrightarrow{\Lambda\mathrm{K}}\neq\vec{0}.\ \Sigma\upsilon\nu\varepsilon\pi\acute{\omega}\varsigma\ \theta\alpha\ \acute{\varepsilon}\chi\omega\ x+y+z=0$

$\underline{\Theta\alpha\ \alpha\pi o\delta\varepsilon\acute{\iota}\xi\omega\ o\tau\iota\ \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \eta\ \sigma\upsilon\nu\varepsilon\pi\alpha\gamma\omega\gamma\acute{\eta}:(2),(3)\Rightarrow(1)}$

$$\acute{E}\chi\omega:\left\{\begin{array}{l} x\overrightarrow{\Lambda\mathrm{A}}+y\overrightarrow{\Lambda\mathrm{B}}+z\overrightarrow{\Lambda\Gamma}=\vec{0} \\ x+y+z=0\end{array}\right\}$$

$$\Theta\acute{\varepsilon}\tau\omega:\boxed{\overrightarrow{\Lambda\mathrm{A}}=\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{KA}},\overrightarrow{\Lambda\mathrm{B}}=\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{KB}},\overrightarrow{\Lambda\Gamma}=\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{K\Gamma}}}$$

$$x\overrightarrow{\Lambda\mathrm{A}}+y\overrightarrow{\Lambda\mathrm{B}}+z\overrightarrow{\Lambda\Gamma}=\vec{0}\ \overset{\substack{\overrightarrow{\Lambda\mathrm{A}}=\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{KA}}\\ \overrightarrow{\Lambda\mathrm{B}}=\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{KB}}\\ \overrightarrow{\Lambda\Gamma}=\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{K\Gamma}}}}{\Longrightarrow}$$

$$x\left(\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{KA}}\right)+y\left(\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{KB}}\right)+z\left(\overrightarrow{\Lambda\mathrm{K}}+\overrightarrow{\mathrm{K\Gamma}}\right)=\vec{0}\Rightarrow$$

$$x\overrightarrow{\Lambda\mathrm{K}}+x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\Lambda\mathrm{K}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\Lambda\mathrm{K}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\Rightarrow$$

$$x\overrightarrow{\Lambda\mathrm{K}}+y\overrightarrow{\Lambda\mathrm{K}}+z\overrightarrow{\Lambda\mathrm{K}}+x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\Rightarrow$$

$$\left(x+y+z\right)\overrightarrow{\mathrm{\Lambda K}}+x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\overset{x+y+z=0}{\Longrightarrow}$$

$$0\bullet\overrightarrow{\mathrm{\Lambda K}}+x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\overset{0\bullet\overrightarrow{\mathrm{\Lambda K}}=\vec{0}}{\Longrightarrow}$$

$$\vec{0}+x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\Rightarrow x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}$$

$\underline{\Theta\alpha\ \alpha\pi o\delta\varepsilon\acute{\iota}\xi\omega\ o\tau\iota\ \iota\sigma\chi\acute{\upsilon}\varepsilon\iota\ \eta\ \sigma\upsilon\nu\varepsilon\pi\alpha\gamma\omega\gamma\acute{\eta}:(1),(3)\Rightarrow(2)}$

$$\acute{E}\chi\omega:\begin{Bmatrix}x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\\ x+y+z=0\end{Bmatrix}$$

$$\Theta\acute{\varepsilon}\tau\omega:\boxed{\overrightarrow{\mathrm{KA}}=\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{\Lambda A}},\overrightarrow{\mathrm{KB}}=\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{KB}},\overrightarrow{\mathrm{K\Gamma}}=\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{\Lambda\Gamma}}}$$

$$x\overrightarrow{\mathrm{KA}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Gamma}}=\vec{0}\overset{\substack{\overrightarrow{\mathrm{KA}}=\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{\Lambda A}}\\ \overrightarrow{\mathrm{KB}}=\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{KB}}\\ \overrightarrow{\mathrm{K\Gamma}}=\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{\Lambda\Gamma}}}}{\Longrightarrow}$$

$$x\left(\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{\Lambda A}}\right)+y\left(\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{KB}}\right)+z\left(\overrightarrow{\mathrm{K\Lambda}}+\overrightarrow{\mathrm{\Lambda\Gamma}}\right)=\vec{0}\Rightarrow$$

$$x\overrightarrow{\mathrm{K\Lambda}}+x\overrightarrow{\mathrm{\Lambda A}}+y\overrightarrow{\mathrm{K\Lambda}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{K\Lambda}}+z\overrightarrow{\mathrm{\Lambda\Gamma}}=\vec{0}\Rightarrow$$

$$x\overrightarrow{\mathrm{K\Lambda}}+y\overrightarrow{\mathrm{K\Lambda}}+z\overrightarrow{\mathrm{K\Lambda}}+x\overrightarrow{\mathrm{\Lambda A}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{\Lambda\Gamma}}=\vec{0}$$

$$\left(x+y+z\right)\overrightarrow{\mathrm{K\Lambda}}+x\overrightarrow{\mathrm{\Lambda A}}+y\overrightarrow{\mathrm{KB}}+z\overrightarrow{\mathrm{\Lambda\Gamma}}=\vec{0}\overset{x+y+z=0}{\Longrightarrow}$$

$$0\bullet\overrightarrow{\mathrm{K\Lambda}}+x\overrightarrow{\mathrm{\Lambda A}}+y\overrightarrow{\mathrm{\Lambda B}}+z\overrightarrow{\mathrm{\Lambda\Gamma}}=\vec{0}\overset{0\bullet\overrightarrow{\mathrm{K\Lambda}}=\vec{0}}{\Longrightarrow}$$

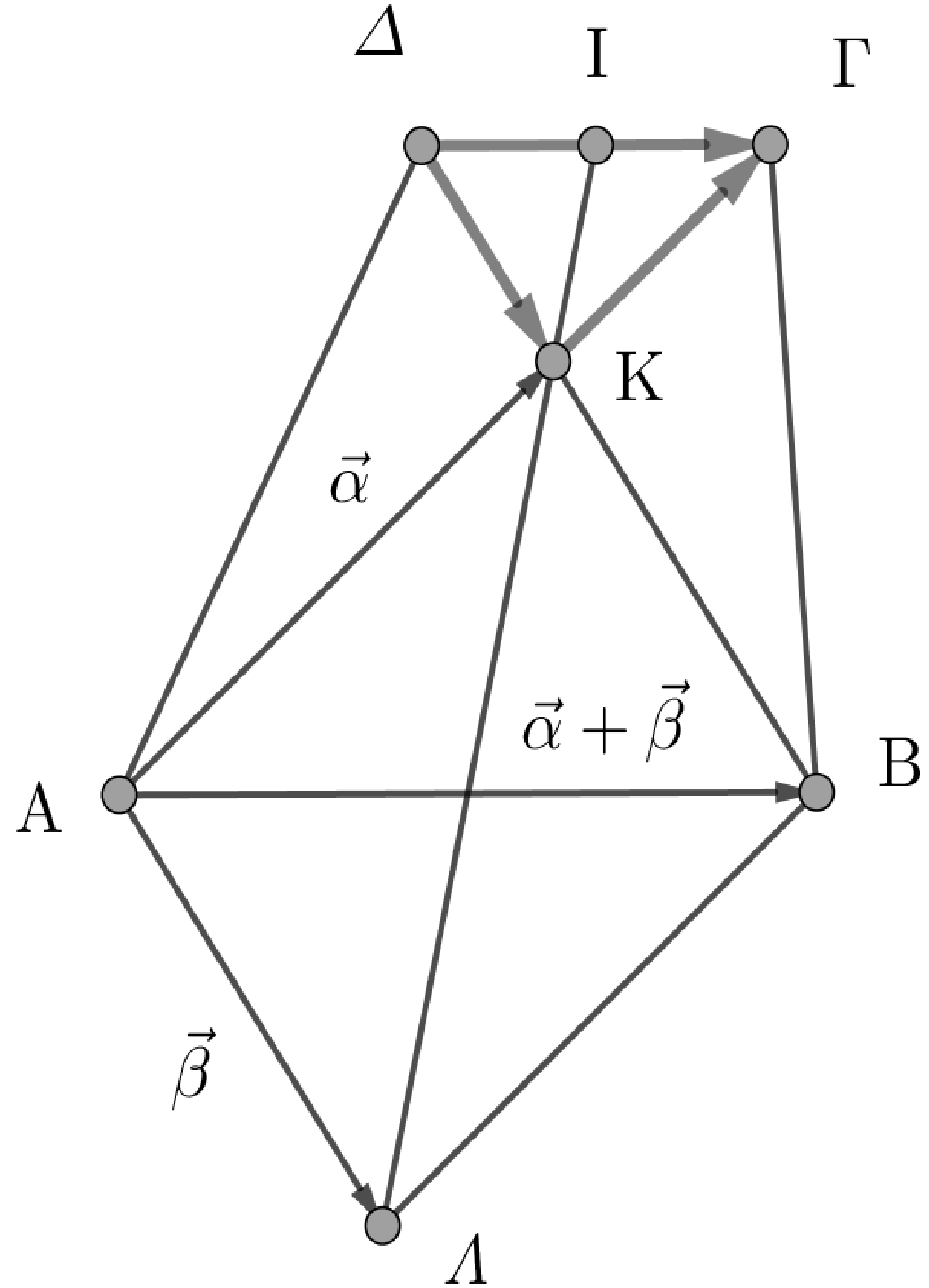
$$\vec{0}+x\overrightarrow{\mathrm{\Lambda A}}+y\overrightarrow{\mathrm{\Lambda B}}+z\overrightarrow{\mathrm{\Lambda\Gamma}}=\vec{0}\Rightarrow x\overrightarrow{\mathrm{\Lambda A}}+y\overrightarrow{\mathrm{\Lambda B}}+z\overrightarrow{\mathrm{\Lambda\Gamma}}=\vec{0}$$

7.

Στο παρακάτω σχήμα το $\mathrm{AB\Gamma\Delta}$ *είναι τραπέζιο με* $\left(\mathrm{AB}\right)=2\left(\Gamma\Delta\right)$, *το* $\mathrm{KA\Lambda B}$ *παραλληλόγραμμο και* I *το μέσο του* $\Gamma\Delta$. *Να αποδείξετε ότι:*

$(\mathrm{I})\ \overrightarrow{\mathrm{K\Gamma}}=-\dfrac{1}{2}\overrightarrow{\mathrm{KA}}$ *και* $\overrightarrow{\mathrm{K\Delta}}=-\dfrac{1}{2}\overrightarrow{\mathrm{KB}}$

(II) *Τα σημεία* $\mathrm{I},\mathrm{K},\Lambda$ *είναι συνευθειακά*



Θέτω: $\boxed{\overrightarrow{\mathrm{AK}}=\vec{\alpha},\overrightarrow{\mathrm{A\Lambda}}=\vec{\beta}}$

Επειδή ΚΑΛΒ *παραλληλόγραμμο θα έχω*:

$$\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{AK}}+\overrightarrow{\mathrm{A\Lambda}}\xRightarrow[\overrightarrow{\mathrm{A\Lambda}}=\vec{\beta}]{\overrightarrow{\mathrm{AK}}=\vec{\alpha}}\boxed{\overrightarrow{\mathrm{AB}}=\vec{\alpha}+\vec{\beta}}$$

$\text{Επειδή } \overrightarrow{\Delta\Gamma} \nearrow\nearrow \overrightarrow{AB} \text{ και } \Delta\Gamma = \frac{AB}{2} \text{ θα έχω:}$

$$\overrightarrow{\Delta\Gamma} = \frac{\overrightarrow{AB}}{2} \xRightarrow{\overrightarrow{AB}=\vec{\alpha}+\vec{\beta}} \boxed{\overrightarrow{\Delta\Gamma} = \frac{\vec{\alpha}+\vec{\beta}}{2}}$$

$\text{Επειδή } \overrightarrow{K\Delta} // \overrightarrow{KB} \text{ θα υπάρχει } \mu \in \mathbb{R} \text{ τέτοιο ώστε να ισχύει:}$

$$\overrightarrow{K\Delta} = \mu\overrightarrow{KB}\left(\begin{array}{l}\text{Έχω } \overrightarrow{KB} = \overrightarrow{A\Lambda} \text{ γιατί το } AKB\Lambda \text{ είναι} \\ \text{παραλληλόγραμμο}\end{array}\right)$$

$$\overrightarrow{K\Delta} = \mu\overrightarrow{A\Lambda} \xRightarrow{\overrightarrow{A\Lambda}=\vec{\beta}} \boxed{\overrightarrow{K\Delta} = \mu\vec{\beta}}$$

$\text{Επειδή } \overrightarrow{K\Gamma} // \overrightarrow{KA} \text{ θα υπάρχει } \lambda \in \mathbb{R} \text{ τέτοιο ώστε να ισχύει:}$

$$\overrightarrow{K\Gamma} = \lambda\overrightarrow{KA} \xRightarrow{\overrightarrow{KA}=-\overrightarrow{AK}} \overrightarrow{K\Gamma} = \lambda\left(-\overrightarrow{AK}\right) \Rightarrow \overrightarrow{K\Gamma} = -\lambda\overrightarrow{AK} \xRightarrow{\overrightarrow{AK}=\vec{\alpha}}$$

$$\boxed{\overrightarrow{K\Gamma} = -\lambda\vec{\alpha}}$$

$\Sigma\text{το τρίγωνο } \Delta K\Gamma \text{ θα έχω:}$

$$\overrightarrow{\Delta\Gamma} = \overrightarrow{\Delta K} + \overrightarrow{K\Gamma} \xLeftrightarrow{\overrightarrow{\Delta K}=-\overrightarrow{K\Delta}} \overrightarrow{\Delta\Gamma} = -\overrightarrow{K\Delta} + \overrightarrow{K\Gamma} \overset{\substack{\overrightarrow{\Delta\Gamma}=\frac{\vec{\alpha}+\vec{\beta}}{2} \\ \overrightarrow{K\Delta}=\mu\vec{\beta} \\ \overrightarrow{K\Gamma}=-\lambda\vec{\alpha}}}{\Longleftrightarrow}$$

$$\frac{\vec{\alpha}+\vec{\beta}}{2} = -\mu\vec{\beta} - \lambda\vec{\alpha} \Leftrightarrow \vec{\alpha}+\vec{\beta} = 2\left(-\mu\vec{\beta} - \lambda\vec{\alpha}\right) \Leftrightarrow$$

$$\vec{\alpha}+\vec{\beta} = -2\mu\vec{\beta} - 2\lambda\vec{\alpha} \Leftrightarrow \vec{\alpha} + 2\lambda\vec{\alpha} + \vec{\beta} + 2\mu\vec{\beta} = \vec{0} \Leftrightarrow$$

$$\left(1+2\lambda\right)\vec{\alpha} + \left(1+2\mu\right)\vec{\beta} = \vec{0}\left(1\right)$$

$$\boxed{\begin{array}{l}\text{Αν } \vec{x} \not\parallel \vec{y} \text{ τότε ισχύει:} \\ \kappa\vec{x} + \lambda\vec{y} = \vec{0} \Leftrightarrow \kappa = \lambda = 0\end{array}}$$

$\text{Επειδή } \vec{\alpha} \not\parallel \vec{\beta} \text{ απο την σχέση } (1) \text{ θα έχω:}$

$$\left\{\begin{array}{l}1+2\lambda = 0 \\ 1+2\mu = 0\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}2\lambda = -1 \\ 2\mu = -1\end{array}\right\} \Leftrightarrow \left\{\begin{array}{l}\lambda = -\frac{1}{2} \\ \mu = -\frac{1}{2}\end{array}\right\}$$

$$\overrightarrow{\mathrm{K\Delta}} = \mu\overrightarrow{\mathrm{KB}} \overset{\mu=-\frac{1}{2}}{=} -\frac{1}{2}\overrightarrow{\mathrm{KB}}$$

$$\overrightarrow{\mathrm{K\Gamma}} = \lambda\overrightarrow{\mathrm{KA}} \overset{\mu=-\frac{1}{2}}{=} -\frac{1}{2}\overrightarrow{\mathrm{KA}}$$

$$(\mathrm{II})\overrightarrow{\mathrm{K\Delta}} = \mu\vec{\beta} \overset{\mu=-\frac{1}{2}}{=} -\frac{1}{2}\vec{\beta}$$

$$\overrightarrow{\mathrm{K\Gamma}} = -\lambda\vec{\alpha} \overset{\lambda=-\frac{1}{2}}{=} -\left(-\frac{1}{2}\right)\vec{\alpha} = \frac{1}{2}\vec{\alpha}$$

Επειδή I μέσο της $\Delta\Gamma$ στο τρίγωνο $\mathrm{KI\Gamma}$ θα έχω:

$$\overrightarrow{\mathrm{KI}} = \frac{1}{2}\left(\overrightarrow{\mathrm{K\Delta}} + \overrightarrow{\mathrm{K\Gamma}}\right) \overset{\substack{\overrightarrow{\mathrm{K\Delta}}=-\frac{1}{2}\vec{\beta} \\ \overrightarrow{\mathrm{K\Gamma}}=\frac{1}{2}\vec{\alpha}}}{=} \frac{1}{2}\left(-\frac{1}{2}\vec{\beta} + \frac{1}{2}\vec{\alpha}\right) = \frac{1}{2}\bullet\frac{1}{2}\left(\vec{\alpha} - \vec{\beta}\right) =$$

$$= \frac{1}{4}\left(\vec{\alpha} - \vec{\beta}\right)$$

$$\overrightarrow{\mathrm{K\Lambda}} = \overrightarrow{\mathrm{A\Lambda}} - \overrightarrow{\mathrm{AK}} \overset{\substack{\overrightarrow{\mathrm{A\Lambda}}=\vec{\beta} \\ \overrightarrow{\mathrm{AK}}=\vec{\alpha}}}{=} \vec{\beta} - \vec{\alpha}$$

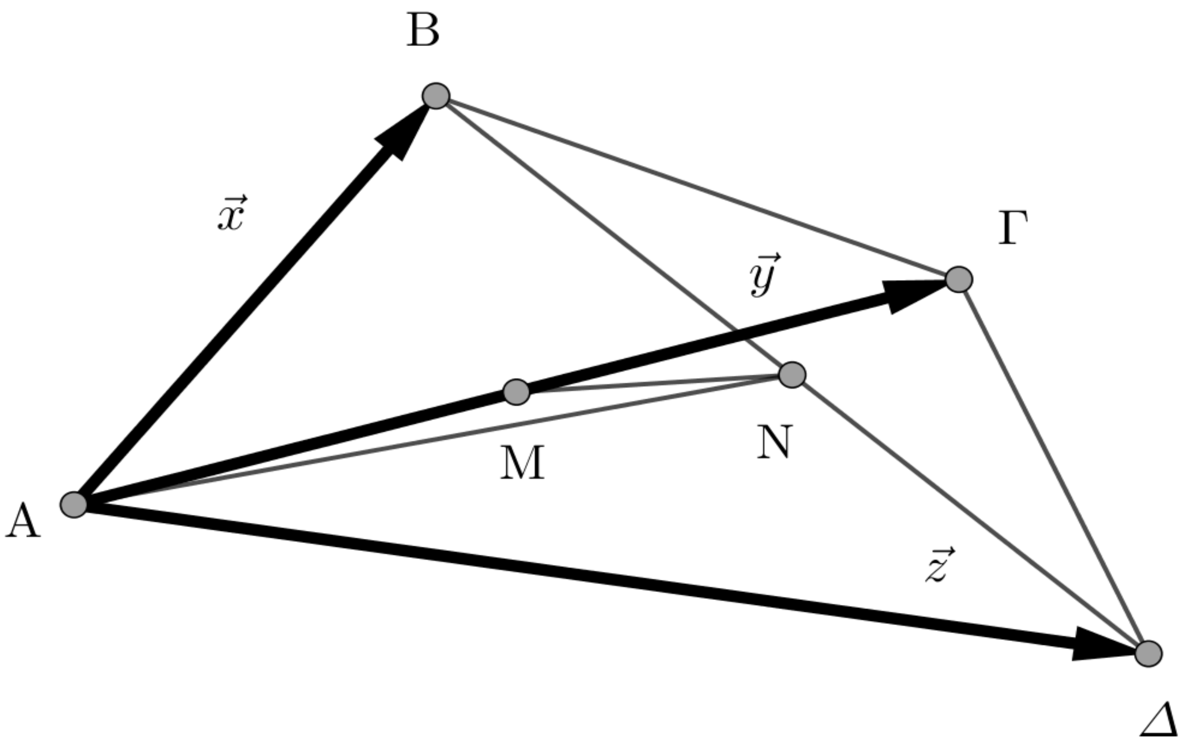
$$\overrightarrow{\mathrm{KI}} = \frac{1}{4}\left(\vec{\alpha} - \vec{\beta}\right) = \frac{1}{4}\left[-\left(\vec{\beta} - \vec{\alpha}\right)\right] = -\frac{1}{4}\left(\vec{\beta} - \vec{\alpha}\right) \overset{\overrightarrow{\mathrm{K\Lambda}}=\vec{\beta}-\vec{\alpha}}{=}$$

$$-\frac{1}{4}\overrightarrow{\mathrm{K\Lambda}}$$

Επειδή $\overrightarrow{\mathrm{KI}} = -\frac{1}{4}\overrightarrow{\mathrm{K\Lambda}}$ προκύπτει ότι $\overrightarrow{\mathrm{KI}} // \overrightarrow{\mathrm{K\Lambda}}$. Συνεπώς τα σημεία $\mathrm{I}, \mathrm{K}, \Lambda$ είναι συνευθειακά

7.

Δίνεται τετράπλευρο $\mathrm{AB\Gamma\Delta}$ και έστω M και N τα μέσα των διαγωνίων του $\mathrm{A\Gamma}$ και $\mathrm{B\Delta}$ αντιστοίχως. Να αποδείξετε ότι αν $4\,\overrightarrow{\mathrm{MN}} = \overrightarrow{\mathrm{A\Delta}} - \overrightarrow{\mathrm{B\Gamma}}$, τότε το τετράπλευρο αυτό είναι παραλληλόγραμμο.



$Θέτω: \boxed{\overrightarrow{ΑΒ}=\vec{x}, \overrightarrow{ΑΓ}=\vec{y}, \overrightarrow{ΑΔ}=\vec{z}}$

Επειδή Μ *μέσο της* ΑΓ *θα έχω*:

$$\overrightarrow{ΑΜ}=\frac{\overrightarrow{ΑΓ}}{2} \overset{\overrightarrow{ΑΓ}=\vec{y}}{\Longrightarrow} \boxed{\overrightarrow{ΑΜ}=\frac{\vec{y}}{2}}$$

Στο *τρίγωνο* ΑΒΔ *το* Ν *είναι το μέσο της* ΒΔ. *Άρα θα έχω*:

$$\overrightarrow{ΑΝ}=\frac{\overrightarrow{ΑΒ}+\overrightarrow{ΑΔ}}{2} \overset{\substack{\overrightarrow{ΑΒ}=\vec{x}\\ \overrightarrow{ΑΔ}=\vec{z}}}{\Longrightarrow} \boxed{\overrightarrow{ΑΝ}=\frac{\vec{x}+\vec{z}}{2}}$$

$$\overrightarrow{ΜΝ}=\overrightarrow{ΑΝ}-\overrightarrow{ΑΜ} \overset{\substack{\overrightarrow{ΑΝ}=\frac{\vec{x}+\vec{z}}{2}\\ \overrightarrow{ΑΜ}=\frac{\vec{y}}{2}}}{\Longrightarrow} \overrightarrow{ΜΝ}=\frac{\vec{x}+\vec{z}}{2}-\frac{\vec{y}}{2} \Rightarrow \boxed{\overrightarrow{ΜΝ}=\frac{1}{2}\left(\vec{x}+\vec{z}-\vec{y}\right)}$$

$$\overrightarrow{ΒΓ}=\overrightarrow{ΑΓ}-\overrightarrow{ΑΒ} \overset{\substack{\overrightarrow{ΑΓ}=\vec{y}\\ \overrightarrow{ΑΒ}=\vec{x}}}{\Longrightarrow} \boxed{\overrightarrow{ΒΓ}=\vec{y}-\vec{x}}$$

$$\overrightarrow{\mathrm{A\Delta}}-\overrightarrow{\mathrm{B\Gamma}}=4\,\overrightarrow{\mathrm{MN}} \overset{\substack{\overrightarrow{\mathrm{A\Delta}}=\vec{z}\\ \overrightarrow{\mathrm{B\Gamma}}=\vec{y}-\vec{x}\\ \overrightarrow{\mathrm{MN}}=\frac{1}{2}\left(\vec{x}+\vec{z}-\vec{y}\right)}}{\Longleftrightarrow} 4\frac{1}{2}\left(\vec{x}+\vec{z}-\vec{y}\right)=\vec{z}-\left(\vec{y}-\vec{x}\right)\Leftrightarrow$$

$$2\left(\vec{x}+\vec{z}-\vec{y}\right)=\vec{z}-\vec{y}+\vec{x}\Leftrightarrow 2\vec{x}+2\vec{z}-2\vec{y}=\vec{z}-\vec{y}+\vec{x}$$

$$\overset{\substack{\text{Μεταφέρω όλα τα } \vec{x}\\ \text{στο πρώτο μέλος και}\\ \text{όλα τα } \vec{z},\vec{y} \text{ στο δεύτερο}\\ \text{μέλος}}}{\Longleftrightarrow} 2\vec{x}-\vec{x}=\vec{z}-2\vec{z}-\vec{y}+2\vec{y}\Leftrightarrow \vec{x}=\vec{y}-\vec{z}$$

$$\overset{\substack{\overrightarrow{\mathrm{AB}}=\vec{x}\\ \overrightarrow{\mathrm{A\Gamma}}=\vec{y}\\ \overrightarrow{\mathrm{A\Delta}}=\vec{z}}}{\Longleftrightarrow}\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{A\Gamma}}-\overrightarrow{\mathrm{A\Delta}} \overset{\overrightarrow{\mathrm{A\Gamma}}-\overrightarrow{\mathrm{A\Delta}}=\overrightarrow{\mathrm{\Delta\Gamma}}}{\Longleftrightarrow} \overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{\Delta\Gamma}}$$

Συνεπώς το τετράπλευρο ΑΒΓΔ *είναι παραλληλόγραμμο.*

8.

Δίνεται παραλληλόγραμμο ΑΒΓΔ. Ν*α βρεθεί σημείο* Μ *τέτοιο ώστε, να ισχύει:*

$$\overrightarrow{\mathrm{MA}}+\overrightarrow{\mathrm{MB}}+\overrightarrow{\mathrm{M\Gamma}}=\overrightarrow{\mathrm{M\Delta}}$$

Θ*έτω*: $\boxed{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\overrightarrow{\mathrm{A\Delta}}=\vec{\beta},\overrightarrow{\mathrm{AM}}=\vec{x}}$

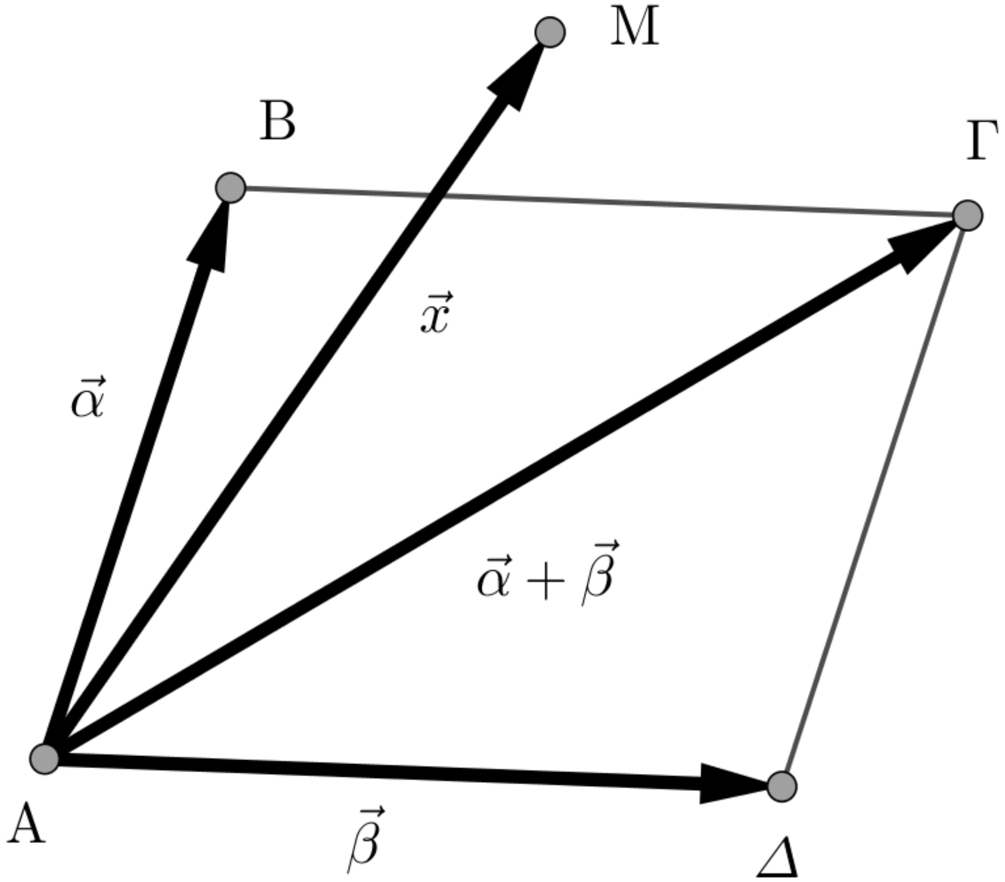
Ε*πειδή* ΑΒΓΔ *παραλληλόγραμμο θα έχω*:

$$\overrightarrow{\mathrm{A\Gamma}}=\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{A\Delta}} \overset{\substack{\overrightarrow{\mathrm{AB}}=\vec{\alpha}\\ \overrightarrow{\mathrm{A\Delta}}=\vec{\beta}}}{\Longrightarrow} \boxed{\overrightarrow{\mathrm{A\Gamma}}=\vec{\alpha}+\vec{\beta}}$$

$$\overrightarrow{\mathrm{MA}}+\overrightarrow{\mathrm{MB}}+\overrightarrow{\mathrm{M\Gamma}}=\overrightarrow{\mathrm{M\Delta}} \overset{\substack{\overrightarrow{\mathrm{MA}}=-\overrightarrow{\mathrm{AM}}\\ \overrightarrow{\mathrm{MB}}=\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{AM}}\\ \overrightarrow{\mathrm{M\Gamma}}=\overrightarrow{\mathrm{A\Gamma}}-\overrightarrow{\mathrm{AM}}\\ \overrightarrow{\mathrm{M\Delta}}=\overrightarrow{\mathrm{A\Delta}}-\overrightarrow{\mathrm{AM}}}}{\Longleftrightarrow}$$

$$-\overrightarrow{\mathrm{AM}}+\overrightarrow{\mathrm{AB}}-\overrightarrow{\mathrm{AM}}+\overrightarrow{\mathrm{A\Gamma}}-\overrightarrow{\mathrm{AM}}=\overrightarrow{\mathrm{A\Delta}}-\overrightarrow{\mathrm{AM}} \overset{\substack{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\\ \overrightarrow{\mathrm{A\Delta}}=\vec{\beta}\\ \overrightarrow{\mathrm{A\Gamma}}=\vec{\alpha}+\vec{\beta}\\ \overrightarrow{\mathrm{AM}}=\vec{x}}}{\Longleftrightarrow}$$

$$-\vec{x}+\vec{a}-\vec{x}+\vec{x}+\cancel{\vec{\beta}}-\cancel{\vec{x}}=\cancel{\vec{\beta}}-\cancel{\vec{x}}\Leftrightarrow -2\vec{x}+2\vec{a}=\vec{0}\Leftrightarrow$$

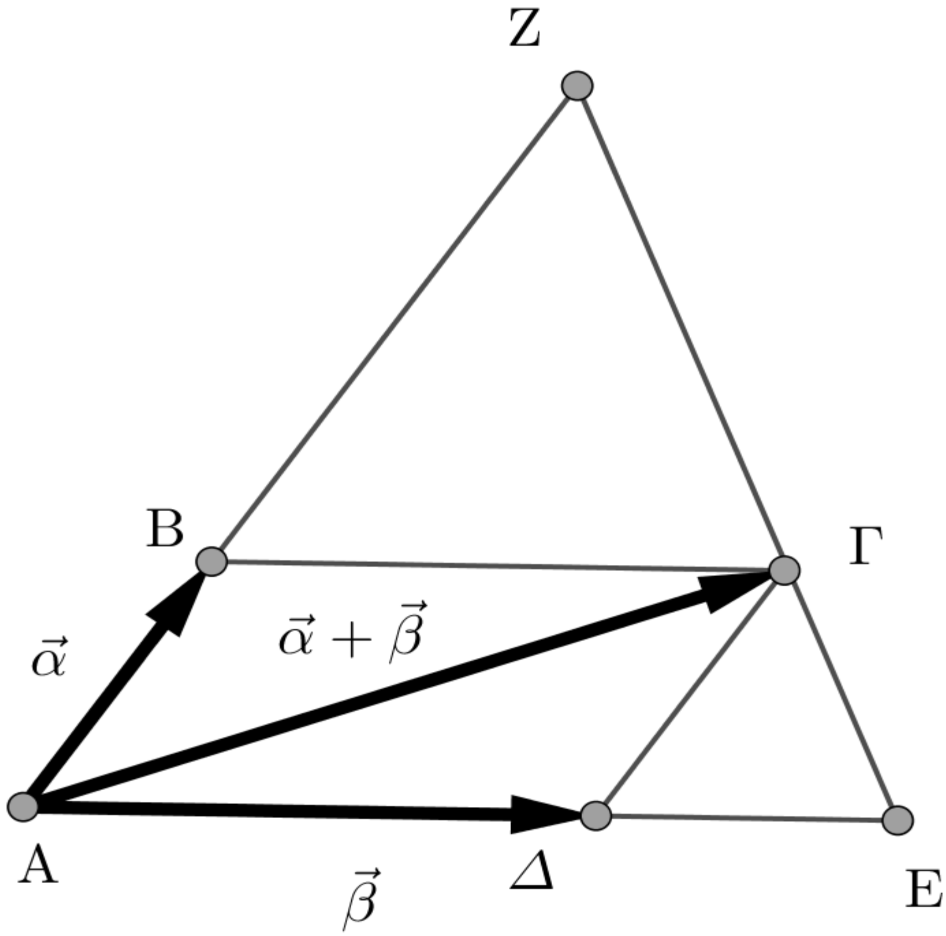


$$2\vec{x}=2\vec{a}\Leftrightarrow\vec{x}=\vec{a}\overset{\substack{\vec{x}=\overrightarrow{\mathrm{AM}}\\ \vec{a}=\overrightarrow{\mathrm{AB}}}}{\Longleftrightarrow}\overrightarrow{\mathrm{AM}}=\overrightarrow{\mathrm{AB}}\Leftrightarrow\overrightarrow{\mathrm{AM}}-\overrightarrow{\mathrm{AB}}=\vec{0}\overset{-\overrightarrow{\mathrm{AB}}=\overrightarrow{\mathrm{BA}}}{\Longleftrightarrow}$$

$$\overrightarrow{\mathrm{BA}}+\overrightarrow{\mathrm{AM}}=\vec{0}\Leftrightarrow\overrightarrow{\mathrm{BM}}=\vec{0}\Leftrightarrow\mathrm{M}\equiv\mathrm{B}$$

9.

Θεωρούμε ένα παραλληλόγραμμο ΑΒΓΔ *και δυο σημεία* Ε *και* Ζ *τέτοια ώστε* $\overrightarrow{\mathrm{AE}}=\kappa\overrightarrow{\mathrm{A\Delta}}$ *και* $\overrightarrow{\mathrm{AZ}}=\lambda\overrightarrow{\mathrm{AB}}$ *όπου* $\lambda=\dfrac{\kappa}{\kappa-1}$ *με* $\kappa\neq1$. *Να αποδείξετε ότι τα σημεία* Ε,Γ,Ζ *είναι συνευθειακά.*



$\Theta\acute{\varepsilon}\tau\omega$: $\boxed{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\ \overrightarrow{\mathrm{A\Delta}}=\vec{\beta}}$

Ε*πειδή* ΑΒΓΔ *παραλληλόγραμμο θα έχω*:

$$\overrightarrow{\mathrm{A\Gamma}}=\overrightarrow{\mathrm{AB}}+\overrightarrow{\mathrm{A\Delta}}\overset{\overrightarrow{\mathrm{AB}}=\vec{\alpha},\ \overrightarrow{\mathrm{A\Delta}}=\vec{\beta}}{\Longrightarrow}\boxed{\overrightarrow{\mathrm{A\Gamma}}=\vec{\alpha}+\vec{\beta}}$$

$$\overrightarrow{\mathrm{AE}}=\kappa\overrightarrow{\mathrm{A\Delta}}\overset{\overrightarrow{\mathrm{A\Delta}}=\vec{\beta}}{\Longrightarrow}\boxed{\overrightarrow{\mathrm{AE}}=\kappa\vec{\beta}}$$

$$\overrightarrow{\mathrm{AZ}}=\lambda\overrightarrow{\mathrm{AB}}\overset{\substack{\lambda=\frac{\kappa}{\kappa-1}\\ \overrightarrow{\mathrm{AB}}=\vec{\alpha}}}{\Longrightarrow}\boxed{\overrightarrow{\mathrm{AZ}}=\frac{\kappa\vec{\alpha}}{\kappa-1}}$$

$$\overrightarrow{\mathrm{Z\Gamma}}=\overrightarrow{\mathrm{A\Gamma}}-\overrightarrow{\mathrm{AZ}}\overset{\substack{\overrightarrow{\mathrm{AZ}}=\frac{\kappa\vec{\alpha}}{\kappa-1}\\ \overrightarrow{\mathrm{A\Gamma}}=\vec{\alpha}+\vec{\beta}}}{=}\vec{\alpha}+\vec{\beta}-\frac{\kappa\vec{\alpha}}{\kappa-1}=\frac{(\kappa-1)\vec{\alpha}+(\kappa-1)\vec{\beta}-\kappa\vec{\alpha}}{\kappa-1}=$$

$$=\frac{(\kappa-1-\kappa)\vec{\alpha}+(\kappa-1)\vec{\beta}}{\kappa-1}=\frac{-\vec{\alpha}+(\kappa-1)\vec{\beta}}{\kappa-1}=\frac{-\left[\vec{\alpha}-(\kappa-1)\vec{\beta}\right]}{\kappa-1}=$$

$$=-\frac{\vec{\alpha}+(1-\kappa)\vec{\beta}}{\kappa-1}=-\frac{1}{\kappa-1}\left[\vec{\alpha}+(1-\kappa)\vec{\beta}\right]=\frac{1}{1-\kappa}\left[\vec{\alpha}+(1-\kappa)\vec{\beta}\right]$$

$\text{Οπότε}: \boxed{\overrightarrow{\mathrm{Z\Gamma}}=\frac{1}{1-\kappa}\left[\vec{\alpha}+(1-\kappa)\vec{\beta}\right]}$

$$\overrightarrow{\mathrm{E\Gamma}}=\overrightarrow{\mathrm{A\Gamma}}-\overrightarrow{\mathrm{AE}}\overset{\substack{\overrightarrow{\mathrm{A\Gamma}}=\vec{\alpha}+\vec{\beta}\\ \overrightarrow{\mathrm{AE}}=\kappa\vec{\beta}}}{=}\vec{\alpha}+\vec{\beta}-\kappa\vec{\beta}=\vec{\alpha}+(1-\kappa)\vec{\beta}$$

$\text{Οπότε}: \boxed{\overrightarrow{\mathrm{E\Gamma}}=\vec{\alpha}+(1-\kappa)\vec{\beta}}$

$$\overrightarrow{\mathrm{Z\Gamma}}=\frac{1}{1-\kappa}\left[\vec{\alpha}+(1-\kappa)\vec{\beta}\right]\overset{\overrightarrow{\mathrm{E\Gamma}}=\vec{\alpha}+(1-\kappa)\vec{\beta}}{=}\frac{1}{1-\kappa}\overrightarrow{\mathrm{E\Gamma}}$$

Επειδή $\overrightarrow{\mathrm{Z\Gamma}}=\frac{1}{1-\kappa}\overrightarrow{\mathrm{E\Gamma}}$ *προκύπτει ότι* $\overrightarrow{\mathrm{Z\Gamma}}\,//\,\overrightarrow{\mathrm{E\Gamma}}$. *Συνεπώς τα σημεία* Ζ,Γ,Ε *είναι συνευθειακά.*

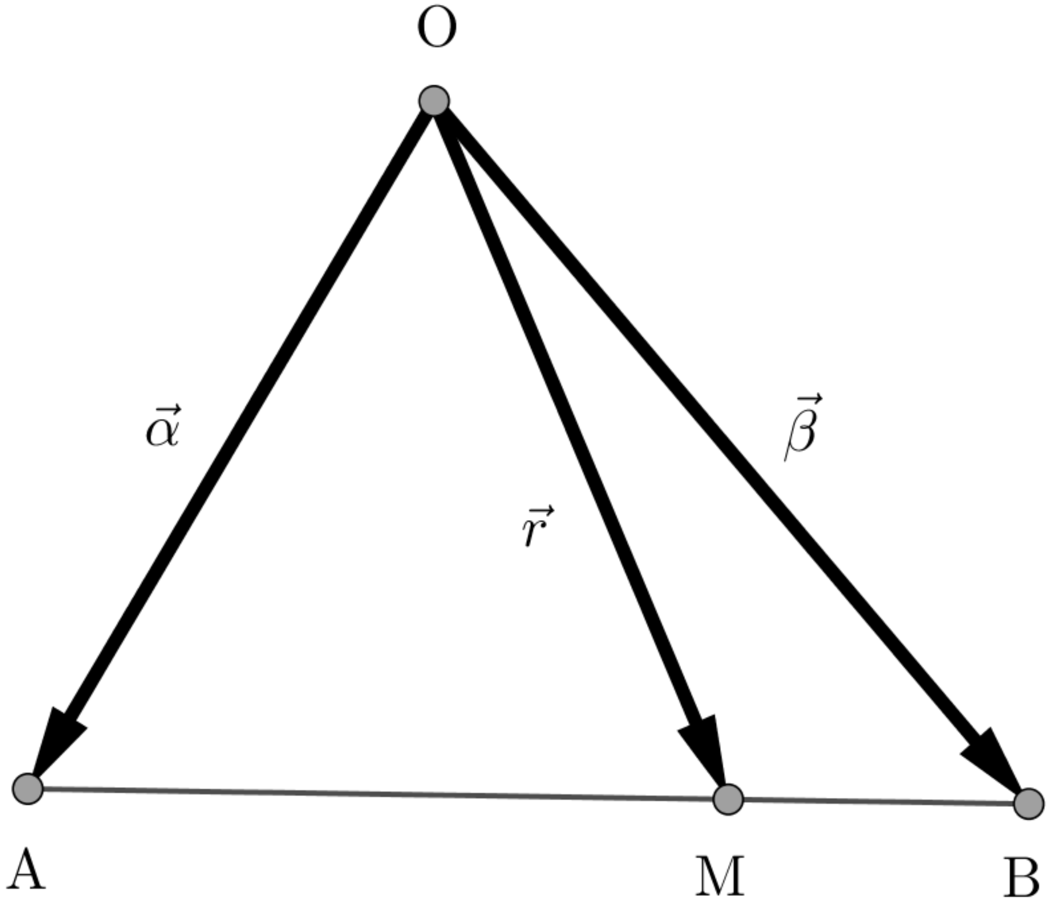
10.

Αν $\vec{\alpha},\vec{\beta}$ *και* $\vec{r}$ *είναι οι διανυσματικές ακτίνες των σημείων* Α,Β *και* Μ *αντιστοίχως και* $\frac{\mathrm{MA}}{\mathrm{MB}}=\frac{\kappa}{\lambda}$ *με* $\kappa,\lambda>0$. *Να αποδείξετε αν το* Μ *είναι εσωτερικό σημείο του* ΑΒ, *τότε ισχύει*: $\vec{r}=\frac{\lambda\vec{\alpha}+\kappa\vec{\beta}}{\lambda+\kappa}$

$\text{Θέτω}: \boxed{\overrightarrow{\mathrm{OA}}=\vec{\alpha},\overrightarrow{\mathrm{OB}}=\vec{\beta},\overrightarrow{\mathrm{OM}}=\vec{r}}$

Επειδή $\overrightarrow{\mathrm{MA}}\nearrow\swarrow\overrightarrow{\mathrm{MB}}$ *και* $\mathrm{MA}=\frac{\kappa}{\lambda}\mathrm{MB}$ *με* $\kappa,\lambda>0$ *θα έχω*:

$$\overrightarrow{\mathrm{MA}}=-\frac{\kappa}{\lambda}\overrightarrow{\mathrm{MB}}\overset{\substack{\overrightarrow{\mathrm{MA}}=\overrightarrow{\mathrm{OA}}-\overrightarrow{\mathrm{OM}}\\ \overrightarrow{\mathrm{MB}}=\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OM}}}}{\Longrightarrow}\overrightarrow{\mathrm{OA}}-\overrightarrow{\mathrm{OM}}=-\frac{\kappa}{\lambda}\left(\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OM}}\right)\overset{\substack{\overrightarrow{\mathrm{OA}}=\vec{\alpha}\\ \overrightarrow{\mathrm{OB}}=\vec{\beta}\\ \overrightarrow{\mathrm{OM}}=\vec{r}}}{\Longrightarrow}$$



$$\vec{\alpha}-\vec{r}=-\frac{\kappa}{\lambda}\left(\vec{\beta}-\vec{r}\right)\Rightarrow\lambda\left(\vec{\alpha}-\vec{r}\right)=-\kappa\left(\vec{\beta}-\vec{r}\right)\Rightarrow$$

$$\lambda\vec{\alpha}-\lambda\vec{r}=-\kappa\vec{\beta}+\kappa\vec{r}\Rightarrow\lambda\vec{\alpha}+\kappa\vec{\beta}=\kappa\vec{r}+\lambda\vec{r}\Rightarrow$$

$$\left(\kappa+\lambda\right)\vec{r}=\lambda\vec{\alpha}+\kappa\vec{\beta}\Rightarrow\vec{r}=\frac{\lambda\vec{\alpha}+\kappa\vec{\beta}}{\kappa+\lambda}$$

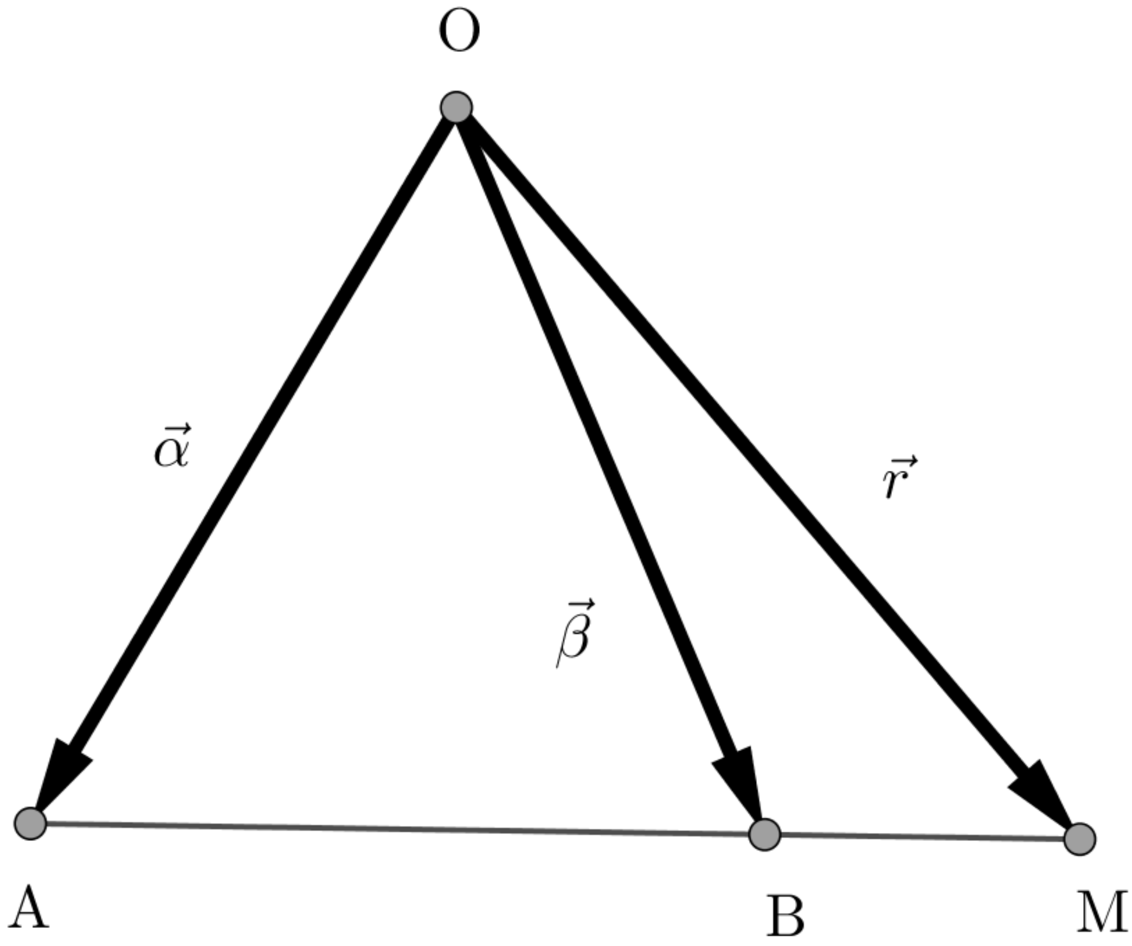
11.

Αν $\vec{\alpha},\vec{\beta}$ *και* $\vec{r}$ *είναι οι διανυσματικές ακτίνες των σημείων*

Α,Β *και* Μ *αντιστοίχως και* $\frac{\text{MA}}{\text{MB}}=\frac{\kappa}{\lambda}$ *με* $\kappa,\lambda>0$ *με* $\kappa\neq\lambda$.

Ν*α αποδείξετε αν το* Μ *είναι εξωτερικό σημείο του* ΑΒ,

τότε ισχύει : $\vec{r}=\dfrac{\lambda\vec{\alpha}-\kappa\vec{\beta}}{\lambda-\kappa}$



$\Theta\acute{\varepsilon}\tau\omega: \boxed{\overrightarrow{\mathrm{OA}} = \vec{\alpha}, \overrightarrow{\mathrm{OB}} = \vec{\beta}, \overrightarrow{\mathrm{OM}} = \vec{r}}$

$\mathrm{E}\pi\varepsilon\iota\delta\acute{\eta}\ \overrightarrow{\mathrm{MA}} \nearrow\!\nearrow \overrightarrow{\mathrm{MB}}\ \kappa\alpha\iota\ \mathrm{MA} = \frac{\kappa}{\lambda}\mathrm{MB}\ \mu\varepsilon\ \kappa, \lambda > 0\ \theta\alpha\ \acute{\varepsilon}\chi\omega:$

$$\overrightarrow{\mathrm{MA}} = \frac{\kappa}{\lambda}\overrightarrow{\mathrm{MB}} \overset{\substack{\overrightarrow{\mathrm{MA}}=\overrightarrow{\mathrm{OA}}-\overrightarrow{\mathrm{OM}} \\ \overrightarrow{\mathrm{MB}}=\overrightarrow{\mathrm{OB}}-\overrightarrow{\mathrm{OM}}}}{\Longrightarrow} \overrightarrow{\mathrm{OA}} - \overrightarrow{\mathrm{OM}} = \frac{\kappa}{\lambda}\left(\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OM}}\right) \overset{\substack{\overrightarrow{\mathrm{OA}}=\vec{\alpha} \\ \overrightarrow{\mathrm{OB}}=\vec{\beta} \\ \overrightarrow{\mathrm{OM}}=\vec{r}}}{\Longrightarrow}$$

$$\vec{\alpha} - \vec{r} = \frac{\kappa}{\lambda}\left(\vec{\beta} - \vec{r}\right) \Rightarrow \lambda\left(\vec{\alpha} - \vec{r}\right) = \kappa\left(\vec{\beta} - \vec{r}\right) \Rightarrow$$

$$\lambda\vec{\alpha} - \lambda\vec{r} = \overrightarrow{\kappa\beta} - \kappa\vec{r} \Rightarrow \lambda\vec{\alpha} - \overrightarrow{\kappa\beta} = \lambda\vec{r} - \kappa\vec{r} \Rightarrow \left(\lambda - \kappa\right)\vec{r} = \lambda\vec{\alpha} - \overrightarrow{\kappa\beta}$$

$$\vec{r} = \frac{\lambda\vec{\alpha} - \overrightarrow{\kappa\beta}}{\lambda - \kappa}$$

12.

Δ *ίνονται τα σημεία* A,B *και* Γ. N*α αποδείξετε ότι για οποιοδήποτε σημείο* M *το διάνυσμα* $3\overrightarrow{MA}-5\overrightarrow{MB}+2\overrightarrow{M\Gamma}$ *παραμένει σταθερό.*

$\Theta\acute{\varepsilon}\tau\omega: \boxed{\overrightarrow{MA}=-\overrightarrow{AM}, \overrightarrow{MB}=\overrightarrow{AB}-\overrightarrow{AM}, \overrightarrow{M\Gamma}=\overrightarrow{A\Gamma}-\overrightarrow{AM}}$

$$3\overrightarrow{MA}-5\overrightarrow{MB}+2\overrightarrow{M\Gamma} \overset{\substack{\overrightarrow{MA}=-\overrightarrow{AM}\\ \overrightarrow{MB}=\overrightarrow{AB}-\overrightarrow{AM}\\ \overrightarrow{M\Gamma}=\overrightarrow{A\Gamma}-\overrightarrow{AM}}}{=}$$

$$3\left(-\overrightarrow{AM}\right)-5\left(\overrightarrow{AB}-\overrightarrow{AM}\right)+2\left(\overrightarrow{A\Gamma}-\overrightarrow{AM}\right)=$$

$$-3\overrightarrow{AM}-5\overrightarrow{AB}+5\overrightarrow{AM}+2\overrightarrow{A\Gamma}-2\overrightarrow{AM}=-5\overrightarrow{AB}+2\overrightarrow{A\Gamma}=$$

$$=\Sigma\tau\alpha\theta\varepsilon\rho\acute{o}$$

13.

Δ *ίνεται το μη μηδενικό διάνυσμα* $\overrightarrow{AB}$ *και σημείο* Γ *τέτοιο ώστε να ισχύει* $\overrightarrow{A\Gamma}=\lambda\overrightarrow{AB}$ *και* $\overrightarrow{B\Gamma}=\mu\overrightarrow{AB}$. N*α αποδείξετε ότι* $\lambda-\mu=1$

$$\acute{E}\chi\omega: \overrightarrow{AB}=\overrightarrow{A\Gamma}+\overrightarrow{\Gamma B} \overset{\overrightarrow{\Gamma B}=-\overrightarrow{B\Gamma}}{\Longleftrightarrow} \overrightarrow{AB}=\overrightarrow{A\Gamma}-\overrightarrow{B\Gamma} \overset{\substack{\overrightarrow{A\Gamma}=\lambda\overrightarrow{AB}\\ \overrightarrow{B\Gamma}=\mu\overrightarrow{AB}}}{\Longleftrightarrow}$$

$$\overrightarrow{AB}=\lambda\overrightarrow{AB}-\mu\overrightarrow{AB} \Leftrightarrow \overrightarrow{AB}-\lambda\overrightarrow{AB}+\mu\overrightarrow{AB}=\vec{0} \Leftrightarrow$$

$$(1-\lambda+\mu)\overrightarrow{AB}=\vec{0}\,(1)$$

Eπειδή $\overrightarrow{AB}\neq\vec{0}$ *απο την σχέση* (1) *θα έχω*:

$$1-\lambda+\mu=0 \Leftrightarrow -\lambda+\mu=-1 \Leftrightarrow -(-\lambda+\mu)=-(-1) \Leftrightarrow$$

$$\lambda-\mu=1$$

14.

A*ν* $\overrightarrow{A\Delta}=\kappa\overrightarrow{AB}+\lambda\overrightarrow{A\Gamma}$ *και* $\overrightarrow{AE}=\lambda\overrightarrow{AB}+\kappa\overrightarrow{A\Gamma}$ *να αποδείξετε ότι* $\overrightarrow{\Delta E}\,//\,\overrightarrow{B\Gamma}$

$$\overrightarrow{\Delta\mathrm{E}} = \overrightarrow{\mathrm{AE}} - \overrightarrow{\mathrm{A}\Delta} \overset{\substack{\overrightarrow{\mathrm{AE}}=\lambda\overrightarrow{\mathrm{AB}}+\kappa\overrightarrow{\mathrm{A}\Gamma} \\ \overrightarrow{\mathrm{A}\Delta}=\kappa\overrightarrow{\mathrm{AB}}+\lambda\overrightarrow{\mathrm{A}\Gamma}}}{=} \lambda\overrightarrow{\mathrm{AB}} + \kappa\overrightarrow{\mathrm{A}\Gamma} - \left(\kappa\overrightarrow{\mathrm{AB}} + \lambda\overrightarrow{\mathrm{A}\Gamma}\right) =$$

$$\lambda\overrightarrow{\mathrm{AB}} + \kappa\overrightarrow{\mathrm{A}\Gamma} - \kappa\overrightarrow{\mathrm{AB}} - \lambda\overrightarrow{\mathrm{A}\Gamma} = -(\kappa-\lambda)\overrightarrow{\mathrm{AB}} + (\kappa-\lambda)\overrightarrow{\mathrm{A}\Gamma} =$$

$$(\kappa-\lambda)\left(\overrightarrow{\mathrm{A}\Gamma} - \overrightarrow{\mathrm{AB}}\right) \overset{\overrightarrow{\mathrm{B}\Gamma}=\overrightarrow{\mathrm{A}\Gamma}-\overrightarrow{\mathrm{AB}}}{=} (\kappa-\lambda)\overrightarrow{\mathrm{B}\Gamma}$$

Ε*πειδή* $\overrightarrow{\Delta\mathrm{E}} = (\kappa-\lambda)\overrightarrow{\mathrm{B}\Gamma}$ *προκύπτει ότι* $\overrightarrow{\Delta\mathrm{E}} // \overrightarrow{\mathrm{B}\Gamma}$